# COMP 330: Sequential Models

Chris Jermaine and Kia Teymourian
Rice University

# So Far, Talked About "iid" Data

Each observation independent

Not always realistic!

Often, data are sequential

- Temperature readings
- Words in a sentence
- Parts of speech
- Stock prices
- Many others!

# “Markov Models”

Ubiquitous in data science

Basic idea:

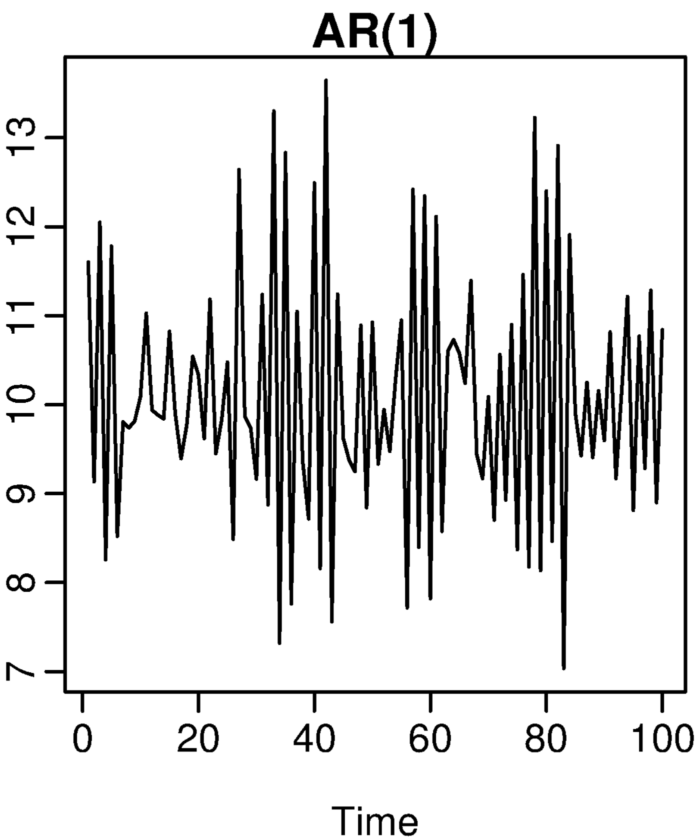
- Data observed at a sequence of “time ticks”
- Data at time tick $t$ is $x_t$
- $x_t$ depends only on $x_{t-1}$
- (Or on $x_{t-d}, x_{t-d+1}, x_{t-d+2}, \ldots, x_{t-1}$ for order-$d$ model)

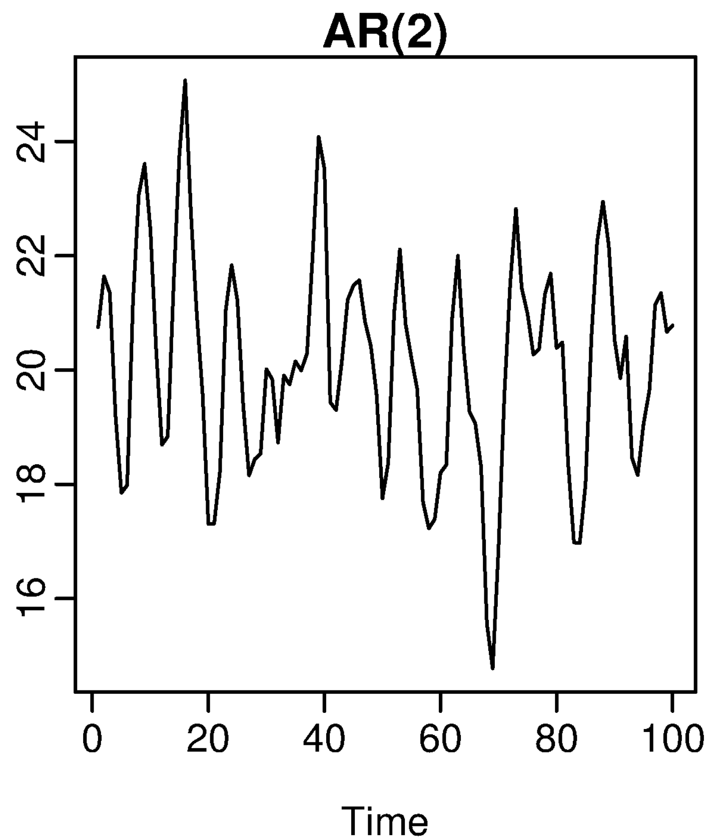
# Classic Sequential Model From Stats

The "Autoregressive" Model

Simple extension of linear regression

- Order-$d$ model is called an AR($d$) model



# Classic Sequential Model From Stats

The "Autoregressive" Model

Simple extension of linear regression

- Order-$d$ model is called an AR($d$) model
- Have $d$ regression coefs for an order-$d$ model
- $r_1, r_2, \ldots, r_d$

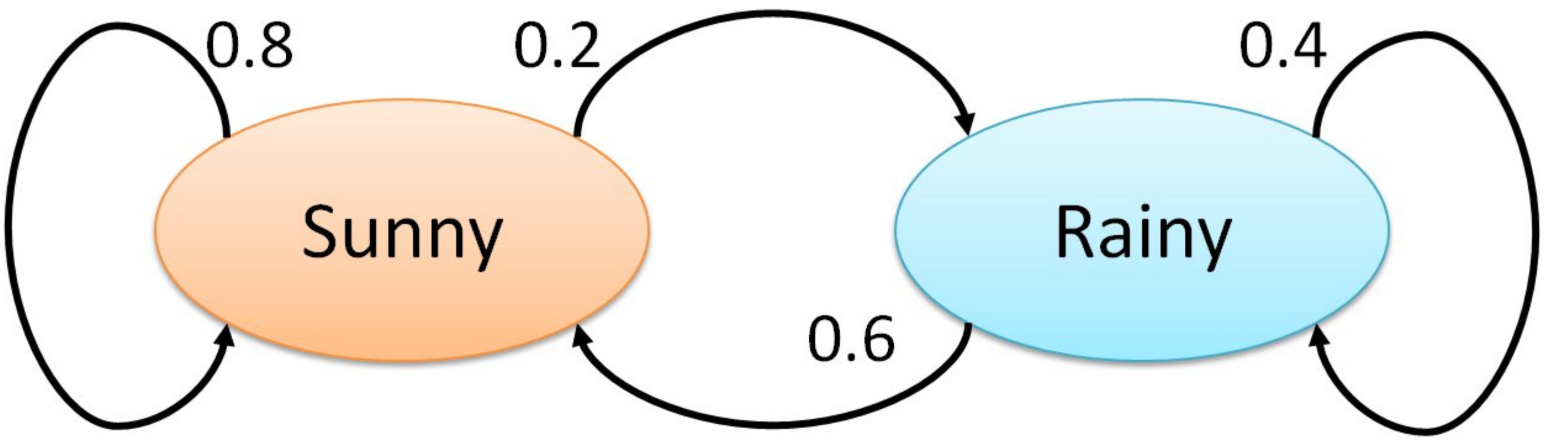
Generative process is:

For $t$ = 1 to $d$ **do:**
  $x_t \sim \text{Normal}(\mu, \sigma^2)$
For $t$ = $d+1$ to $n$ **do:**
  $\theta = \sum_{i=0}^{d-1} r_{i+1} \times x_{t-d+i}$
  $x_t \sim \text{Normal}(\theta, \sigma^2)$

# Classic Sequential Model From CS

## Hidden Markov Model

- Begins with a Markov chain
- Assume that there are $k$ states
- We stochsatically jump around between the states



# Classic Sequential Model From CS

## Hidden Markov Model

- Begins with a Markov chain
- Assume that there are $k$ states
- We stochsatically jump around between the states
- Let $\pi_0$ be probs for start
- Let $\pi_i$ be transition probs out of state $i$

```
s_1 ~ Categorical(π_0)
For t = 2 to n do:
    s_t ~ Categorical(π_{t−1})
```

$s_1 \sim \text{Categorical}(\pi_0)$
For $t$ = 2 to $n$ **do:**
  $s_t \sim \text{Categorical}(\pi_{t-1})$

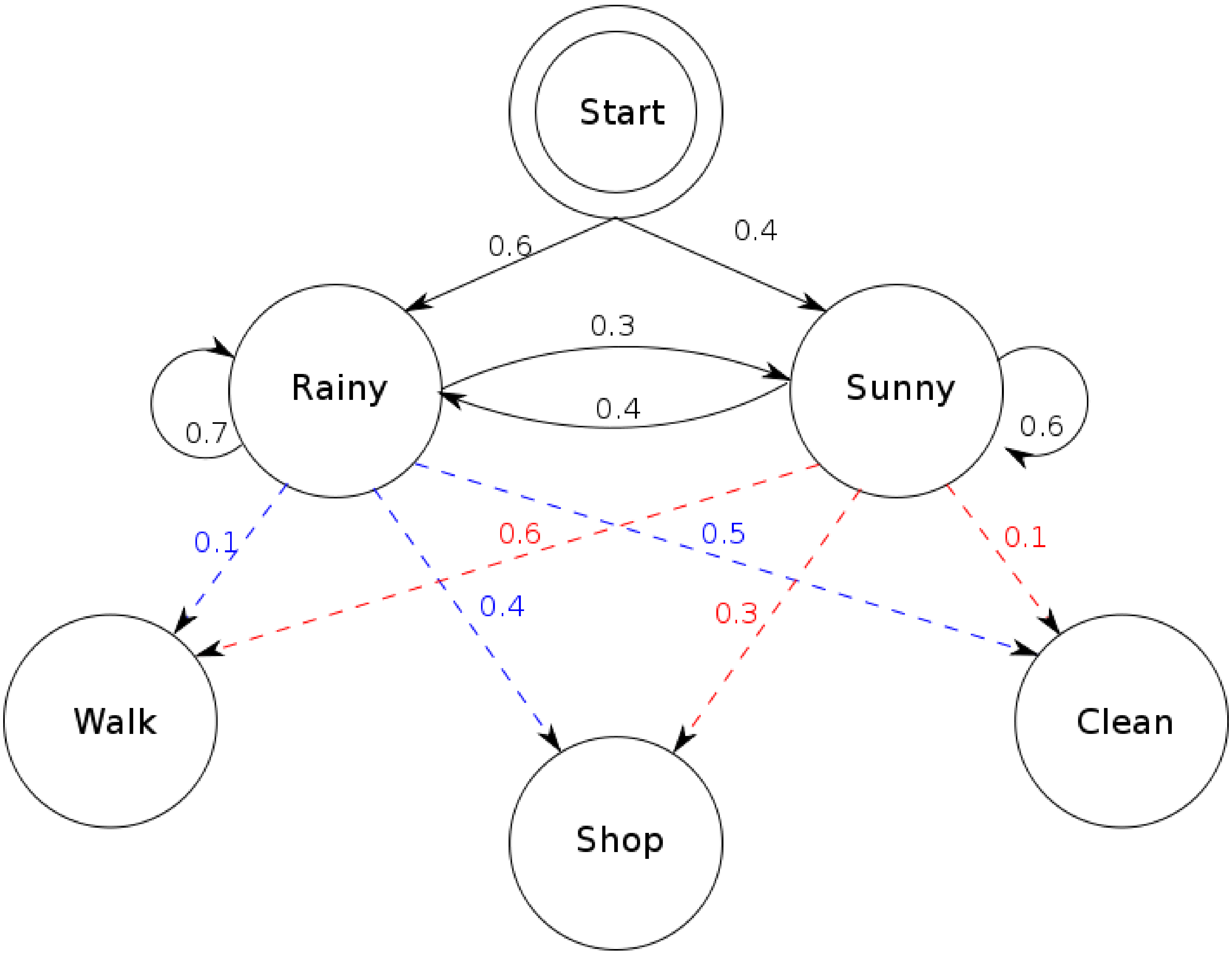
# Classic Sequential Model From CS

Then we add the observed data

- Often Categorical
- Though sometimes not (Normal, Gamma, Poission are common)
- Let $\boldsymbol{\theta}_s$ be parameter set associated with state $\boldsymbol{s}$

$s_1 \sim \text{Categorical}(\pi_0)$
$x_t \sim f(\theta_{s_1})$
For $t$ = 2 to $n$ **do**:
    $s_t \sim \text{Categorical}(\pi_{t-1})$
    $x_t \sim f(\theta_{s_t})$

# Example HMM



Start
0.6
0.4
Rainy
0.3
Sunny
0.4
0.7
0.6
0.1
0.6
0.5
0.1
0.4
0.3
Walk
Clean
Shop

# Now, Let's Derive a Gibbs Sampler!

For a Categorical HMM

Going Bayesian? Need priors on all params

For $j$ = 0 to $k$ **do**:
  $\pi_j \sim \text{Dirichlet}(\alpha)$
For $j$ = 1 to $k$ **do**:
  $\theta_j \sim \text{Dirichlet}(\beta)$

$s_1 \sim \text{Categorical}(\pi_0)$
$x_1 \sim \text{Categorical}(\theta_{s_1})$
For $t$ = 2 to $n$ **do**:
  $s_t \sim \text{Categorical}(\pi_{t-1})$
  $x_t \sim \text{Categorical}(\theta_{s_t})$

# Do the Rest on the Board!

# Questions?