Python, NumPy, and Spark

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Next 1.5 Days

- Intro to Python for statistical/numerical programming (day one)
  - — Focus on basic NumPy API, using arrays efficiently
  - — Will take us through today
- Intro to cloud computing, Big Data computing
  - — Focus on Amazon AWS (but other cloud providers similar)
  - — Focus on Spark: Big Data platform for distributing Python/Java/Scala comps
- Will try to do all of this in context of interesting examples
  - — With a focus on text processing
  - — But ideas applicable to other problems

Python

- Old language, first appeared in 1991
  - — But updated often over the years
- Important characteristics
  - — Interpreted
  - — Dynamically-typed
  - — High level
  - — Multi-paradigm (imperative, functional, OO)
  - — Generally compact, readable, easy-to-use
- Boom on popularity last five years
  - — Now the first PL learned in many CS departments

Python: Why So Popular for Data Science?

- Dynamic typing/interpreted
  - — Type a command, get a result
  - — No need for compile/execute/debug cycle
- Quite high-level: easy for non-CS people to pick up
  - — Statisticians, mathematicians, physicists...
- More of a general-purpose PL than R
  - — More reasonable target for larger applications
  - — More reasonable as API for platforms such as Spark
- Can be used as lightweight wrapper on efficient numerical codes
  - — Unlike Java, for example

First Python Example

- Since Python is interpreted, can just fire up Python shell
  - Then start typing
- Ex, fire up shell and type (exactly!)

```
def Factorial (n):
     if n == 1 or n == 0:
          return 1
     else:
          return n * Factorial (n - 1)

Factorial (12)
```

- Will print out 12 factorial

Python Basics Continued

- Some important Python basics...
- Spacing and indentaton
  - Indentation important... **no** `begin/end` **nor** `{}`... indentation signals code block
  - Blank lines important; can’t have blank line inside of indented code block
- Variables
  - No declaration
  - All type checking dynamic
  - Just use

Python Basics Continued

- Dictionaries
  - — Standard container type is dictionary/map
  - — Example: `wordsInDoc = {}` creates empty dictionary
  - — Add data by saying `wordsInDoc[23] = 16`
  - — Now can write something like `if wordsInDoc[23] == 16:` ...
  - — What if `wordsInDoc[23]` is not there? Will crash
  - — Protect with if `wordsInDoc.get (23, 0)`... returns 0 if key 23 not defined
- Functions/Procedures
  - — Defined using `def myFunc (arg1, arg2):`
  - — Make sure to indent!
  - — Procedure: no `return` statement
  - — Function: `return` statement

Python Basics Continued

- Loops
  - Of form `for var in range (0, 50):`
    loops for var in {0, 1, ..., 49}
  - Or `for var in dataStruct:`
    loops through each entry in `dataStruct`
  - `dataStruct` can be an array, or a dictionary
  - If array, you loop through the entries
  - If dictionary, you loop through the keys
  - Try

```
a = {}
a[1] = ‘this’
a[2] = ‘that’
a[3] = ‘other’
for b in a:
     a[b]
```

NumPy

- NumPy is a Python package
  - Check out `cmj4.web.rice.edu/LDADictionaryBased.html`
- Most important one for data science!
  - Can use it to do super-fast math, statistics
  - Most basic type is NumPy `array`
  - Used to store vectors, matrices, tensors
- You will get some reasonable experience with NumPy
- Load with `import numpy as np`
- Then can say, for example, `np.random.multinomial (numTrials, probVector, numRows)`

NumPy (cont)

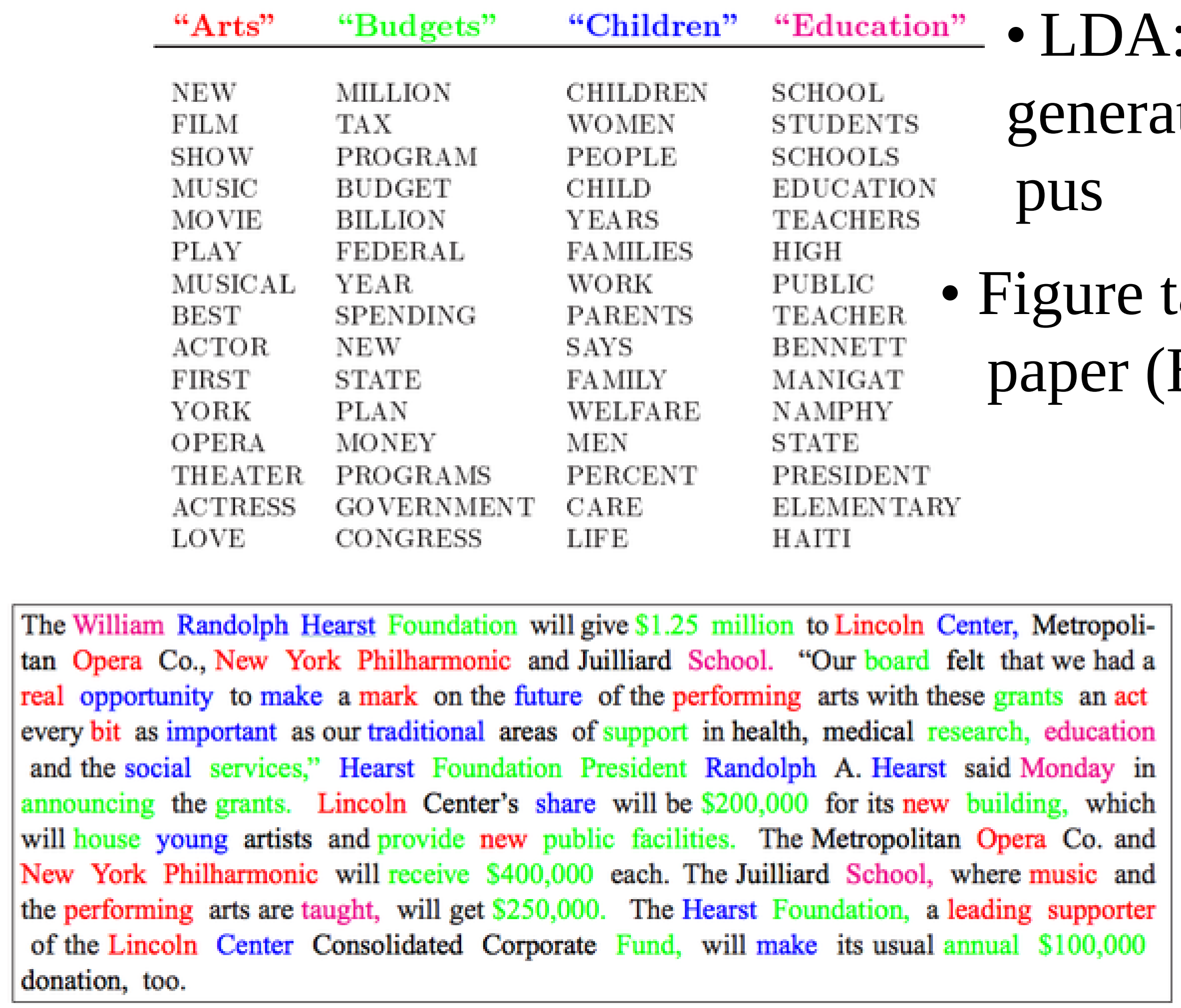
- `np.random.multinomial (numTrials, probVector, numRows)`
  - — Take numRows samples from a Multinomial (probVector, numTrials) dist
- `np.random.multinomial (numTrials, probVector, numRows)`
  - — Take numRows samples from a Multinomial (probVector, numTrials) dist
  - — Put in a matrix with numRows rows
- `np.flatnonzero (array)`
  - — Return array of indices of non-zero elements of array
- `np.random.dirichlet (paramVector, numRows)`
  - — Take numRows samples from a Dirichlet (paramVector) dist
- `np.full (numEntries, val)`
  - — Create a NumPy array with the spec'ed number of entries, all set to val

LDA

— Check out `cmj4.web.rice.edu/LDADictionaryBased.html`

- Don’t want to write code to do something silly
- So we’ll write some code having to do with a commonly-used statistical model for text: “Latent Dirichlet Allocation” or LDA

NumPy Implementation of LDA



"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
PLAY	FEDERAL	FAMILIES	HIGH
MUSICAL	YEAR	WORK	PUBLIC
BEST	SPENDING	PARENTS	TEACHER
ACTOR	NEW	SAYS	BENNETT
FIRST	STATE	FAMILY	MANIGAT
YORK	PLAN	WELFARE	NAMPHY
OPERA	MONEY	MEN	STATE
THEATER	PROGRAMS	PERCENT	PRESIDENT
ACTRESS	GOVERNMENT	CARE	ELEMENTARY
LOVE	CONGRESS	LIFE	HAITI

The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.

Figure 8: An example article from the AP corpus. Each color codes a different factor from which the word is putatively generated.

- LDA: stochastic model for generating a document corpus
- Figure taken from original paper (Blei)

NumPy Implementation of LDA

- Most widely-used “topic model”
- A “topic” is a set of words that appear to gether with high prob
    - — Intuitively: set of words that all have to do with the same subject
- Often, we want to “learn” an LDA model from an existing corpus
    - — But can also use it to generate a corpus
    - — Which we will do today...

LDA Typically Used To Analyze Text

- Idea:
  - — If you can analyze a corpus...
  - — And figure out a set of *k* topics...
  - — As well as how prevalent each topic is in each document
  - — You then know a lot about the corpus
  - — Ex: can use this prevalence info to search the corpus
  - — Two docs have similar topic compositions? Then they are similar!

OK, So What Does This Have To Do W Text?

- Basic LDA setup
  - — LDA will generate *n* random documents given a dictionary
  - — Dictionary is of size num_words
  - — Best shown thru an example
  - — In our example: dictionary will have: (0, “bad”) (1, “I”) (2, “can’t”) (3, “stand”) (4, “comp 215”), (5, “to”) (6, “leave”) (7, “love”) (8, “beer”) (9, “humanities”) (10, “classes”)

LDA Step One

- Generate each of the *k* “topics”
  - — Each topic is represented by a vector of probabilities
  - — The *w*th entry in the vector is associated with the *w*th word in the dictionary
  - — wordsInTopic$_t$[*w*] is the probability that topic *t* would produce word *w*
  - — Vector is sampled from a Dirichlet (alpha) distribution
  - — So, for each *t* in {0...*k* - 1}, wordsInTopic$_t$ ~ Dirichlet (alpha)

LDA Step One

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    - — $\text{wordsInTopic}_t[w]$ is the probability that topic *t* would produce word *w*
    - — Vector is sampled from a Dirichlet (alpha) distribution
    - — So, for each *t* in {0...*k* - 1}, $\text{wordsInTopic}_t \sim$ Dirichlet (alpha)
- Ex: *k* = 3
    - — $\text{wordsInTopic}_0 =$ (.2, .2, .2, .2, 0, 0, 0, 0, .2, 0, 0)
    - — $\text{wordsInTopic}_1 =$ (0, .2, .2, .2, 0, 0, 0, 0, 0, .2, .2)
    - — $\text{wordsInTopic}_2 =$ (0, .2, .2, 0, .2, 0, .2, .2, 0, 0, 0)

LDA Step Two

- Generate the topic proportions for each document
  - — Each topic “controls” a subset of the words in a document
  - — topicsInDoc$_d$[t] is the probability that an arbitrary word in document d will be controlled by topic t
  - — Vector is sampled from a Dirichlet (beta) distribution
  - — So, for each d in {0...n - 1}, topicsInDoc$_d$ ~ Dirichlet (beta)

LDA Step Two

- Generate the topic proportions for each document
  - — Each topic “controls” a subset of the words in a document
  - — $\text{topicsInDoc}_d[t]$ is the probability that an arbitrary word in document *d* will be controlled by topic *t*
  - — Vector is sampled from a Dirichlet (beta) distribution
  - — So, for each *d* in {0...*n* - 1}, topicsInDoc_d ~ Dirichlet (beta)
- Ex: *n* = 4
  - — topicsInDoc_0 = (.98, 0.01, 0.01)
  - — topicsInDoc_1 = (0.01, .98, 0.01)
  - — topicsInDoc_2 = (0.02. .49, .49)
  - — topicsInDoc_3 = (.98, 0.01, 0.01)

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
  - — wordsInDoc$_d[w]$ is the number of occurences of word w in document d
  - — To get this vector, generate the words one-at-a-time
  - — For a given word in doc d:

    (1) Figure out the topic t that controls it by sampling from a Multinomial (topicsInDoc$_d$, 1) distribution

    (2) Generate the word by sampling from a Multinomial (wordsInTopic$_t$, 1) distribution

LDA Step Three

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    - (1) Figure out the topic *t* that controls it by sampling from a Multinomial (topicsInDoc_d, 1) distribution
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- Ex: doc 0... topicsInDoc_0 = (.98, 0.01, 0.01)
  - — *t* for word zero is...

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
  - — $\text{wordsInDoc}_d[w]$ is the number of occurences of word *w* in document *d*
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    Multinomial (wordsInTopic_t, 1) distribution

- Ex: doc 0... topicsInDoc_0 = = (.98, 0.01, 0.01)
  - — *t* for word zero is zero, since we sampled (1, 0, 0) [there is a 1 in the zeroth entry]
  - — So we generate the word using wordsInTopic_0 = (.2, .2, .2, .2, 0, 0, 0, 0, .2, 0, 0)

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
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    (2) Generate the word by sampling from a Multinomial (wordsInTopic_t, 1) distribution
- Ex: doc 0... topicsInDoc_0 = = (.98, 0.01, 0.01) “I”
  - — t for word zero is zero, since we sampled (1, 0, 0) [there is a 1 in the zeroth entry]
  - — So we generate the word using wordsInTopic_0 = (.2, .2, .2, .2, 0, 0, 0, 0, .2, 0, 0)
  - — And we get (0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0), which is equivalent to “I”

LDA Step Three

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- Ex: doc 0... topicsInDoc$_0$ = = (.98, 0.01, 0.01) “I”
  - — Now onto the next word

LDA Step Three

- Generate the words in each document
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    Multinomial (wordsInTopic$_t$, 1) distribution

- Ex: doc 0... topicsInDoc$_0$ = = (.98, 0.01, 0.01) "I"
  - — t for word one is zero, since we sampled (1, 0, 0) [there is a 1 in the zeroth entry]

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
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    (2) Generate the word by sampling from a
    Multinomial (wordsInTopic$_t$, 1) distribution

- Ex: doc 0... topicsInDoc$_0$ = = (.98, 0.01, 0.01) “I can’t”
  - — t for word one is zero, since we sampled (1, 0, 0) [there is a 1 in the zeroth entry]
  - — So we generate the word using wordsInTopic$_0$ = (.2, .2, .2, .2, 0, 0, 0, 0, .2, 0, 0)
  - — And we get (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0), which is equivalent to “can’t”

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
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  - — For a given word in doc *d*:
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    - (2) Generate the word by sampling from a Multinomial (wordsInTopic$_t$, 1) distribution
- Ex: doc 0... topicsInDoc$_0$ = = (.98, 0.01, 0.01) “I can’t”
  - — Now onto the next word

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
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    - (1) Figure out the topic t that controls it by sampling from a Multinomial (topicsInDoc$_d$, 1) distribution
    - (2) Generate the word by sampling from a Multinomial (wordsInTopic$_t$, 1) distribution
- Ex: doc 0... topicsInDoc$_0$ = (.98, 0.01, 0.01) “I can’t”
  - — t for word two is zero, since we sampled (1, 0, 0) [there is a 1 in the zeroth entry]

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
  - — wordsInDoc$_d$[w] is the number of occurences of word w in document d
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  - — For a given word in doc d:
    - (1) Figure out the topic t that controls it by sampling from a Multinomial (topicsInDoc$_d$, 1) distribution
    - (2) Generate the word by sampling from a Multinomial (wordsInTopic$_t$, 1) distribution
- Ex: doc 0... topicsInDoc$_0$ = (.98, 0.01, 0.01) “I can’t stand”
  - — t for word two is zero, since we sampled (1, 0, 0) [there is a 1 in the zeroth entry]
  - — So we generate the word using wordsInTopic$_0$ = (.2, .2, .2, .2, 0, 0, 0, 0, .2, 0, 0)
  - — And we get (0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0), which is equivalent to “stand”

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
  - — wordsInDoc$_d$[w] is the number of occurences of word w in document d
  - — To get this vector, generate the words one-at-a-time
  - — For a given word in doc d:
    - (1) Figure out the topic t that controls it by sampling from a Multinomial (topicsInDoc$_d$, 1) distribution
    - (2) Generate the word by sampling from a Multinomial (wordsInTopic$_t$, 1) distribution
- Ex: doc 0... topicsInDoc$_0$ = (.98, 0.01, 0.01) “I can’t stand”
  - — Onto next word

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
  - — wordsInDoc$_d$[w] is the number of occurences of word w in document d
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  - — For a given word in doc d:
    - (1) Figure out the topic t that controls it by sampling from a Multinomial (topicsInDoc$_d$, 1) distribution
    - (2) Generate the word by sampling from a Multinomial (wordsInTopic$_t$, 1) distribution
- Ex: doc 0... topicsInDoc$_0$ = (.98, 0.01, 0.01) “I can’t stand”
  - — t for word three is zero, since we sampled (1, 0, 0) [there is a 1 in the zeroth entry]

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
  - — wordsInDoc$_d$[*w*] is the number of occurences of word *w* in document *d*
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  - — For a given word in doc *d*:
    - (1) Figure out the topic *t* that controls it by sampling from a Multinomial (topicsInDoc$_d$, 1) distribution
    - (2) Generate the word by sampling from a Multinomial (wordsInTopic$_t$, 1) distribution
- Ex: doc 0... topicsInDoc$_0$ = (.98, 0.01, 0.01) “I can’t stand bad”
  - — *t* for word three is zero, since we sampled (1, 0, 0) [there is a 1 in the zeroth entry]
  - — So we generate the word using wordsInTopic$_0$ = (.2, .2, .2, .2, 0, 0, 0, 0, .2, 0, 0)
  - — And we get (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0), which is equivalent to “bad”

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
  - — $\text{wordsInDoc}_d[w]$ is the number of occurences of word w in document d
  - — To get this vector, generate the words one-at-a-time
  - — For a given word in doc d:
    - (1) Figure out the topic t that controls it by sampling from a Multinomial (topicsInDoc_d, 1) distribution
    - (2) Generate the word by sampling from a Multinomial (wordsInTopic_t, 1) distribution
- Ex: doc 0... topicsInDoc_0 = (.98, 0.01, 0.01) “I can’t stand bad”
  - — Onto the last word in the document

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
  - — wordsInDoc$_d$[w] is the number of occurences of word w in document d
  - — To get this vector, generate the words one-at-a-time
  - — For a given word in doc d:

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    (2) Generate the word by sampling from a Multinomial (wordsInTopic$_t$, 1) distribution
- Ex: doc 0... topicsInDoc$_0$ = (.98, 0.01, 0.01) “I can’t stand bad”
  - — t for word three is zero, since we sampled (1, 0, 0) [there is a 1 in the zeroth entry]

LDA Step Three

- Generate the words in each document
  - — Each topic “controls” a subset of the words in a document
  - — wordsInDoc$_d$[*w*] is the number of occurences of word *w* in document *d*
  - — To get this vector, generate the words one-at-a-time
  - — For a given word in doc *d*:

    (1) Figure out the topic *t* that controls it by sampling from a Multinomial (topicsInDoc$_d$, 1) distribution

    (2) Generate the word by sampling from a Multinomial (wordsInTopic$_t$, 1) distribution

- Ex: doc 0... topicsInDoc$_0$ = (.98, 0.01, 0.01) “I can’t stand bad beer”
  - — *t* for word three is zero, since we sampled (1, 0, 0) [there is a 1 in the zeroth entry]
  - — So we generate the word using wordsInTopic$_0$ = (.2, .2, .2, .2, 0, 0, 0, 0, .2, 0, 0)
  - — And we get (0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0), which is equivalent to “beer”

In The End… For Doc 0…

- text is “I can’t stand bad beer” (equiv. to “1 2 3 0 8”)
- topicsInDoc_0 = (.98, 0.01, 0.01)
- wordsInDoc_0 = (1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0)
  - — Why? Word 0 appears once, word 1 appears once, word 4 zero times, etc.
- produced_0= (1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0)

  (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)

  (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
  - — Why? Topic 0 (associated with first line) produced 5 words

    Those words were (1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0)
  - — Topic 1, topic 2 produced no words
  - — “produced” always a matrix with num_words cols, *k* rows

Repeat For Each Doc in the Corpus!

For Example, Let’s Look At Doc 2…

- topicsInDoc_2 = (.02, 0.49, 0.49)
- Imagine that when we generate doc 2, we get:
  - — Word 0: produced by topic 2, is 1 or “I”
  - — Word 1: produced by topic 2, is 7 or “love”
  - — Word 2: produced by topic 2, is 8 or “beer”
  - — Word 3: produced by topic 1, is 1 or “I”
  - — Word 4: produced by topic 1, is 2 or “can’t”
  - — Word 5: produced by topic 2, is 7 or “love”
  - — Word 6: produced by topic 1, is 9 or “humanities”
  - — Word 7: produced by topic 1, is 10 or “classes”
- wordsInDoc_2 = (0, 2, 1, 0, 0, 0, 0, 2, 1, 1, 1)
- produced_2= (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
  (0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1)
  (0, 1, 0, 0, 0, 0, 0, 2, 1, 0, 0)

OK. Back to Python

— Check out `cmj4.web.rice.edu/LDADictionaryBased.html`

- Can you complete the activity?

Problem: Bad Code!

- No one should write statistical/math Python code this way
- Python is **interpreted**
  - Time for each statement execution generally large
- Fewer statements executed, even if work same == performance
- Goal:
  - Try to replace dictionaries with NumPy arrays
  - Try to replace loops with bulk array operations
  - Backed by efficient, low-level implementations
  - Known as “vectorized” programming

Better Code

- Check out `cmj4.web.rice.edu/LDAArrays.html`
- No dictionaries here! Just arrays.
  - — Can you complete the code?

Advantages of Vectorization

- Co-occurence analysis
  - — fundamental task in many statistical/data mining computations
- In text processing...
  - — Given a document corpus
  - — Want to count number of times (word1, word2) occur in same doc in corpus
- Your task: build three implementations
  - — Utilizing varying degrees of vectorization
  - — We will time each, see which is faster

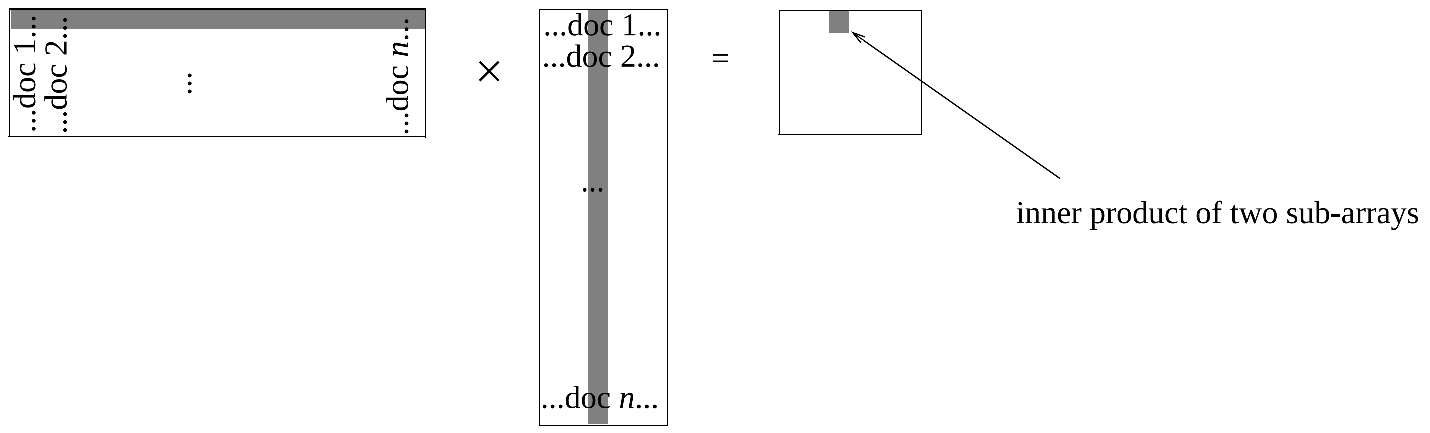
Imp 1: Pure Dictionary-Based

- Pure nested loops implementation
  - Has advantage that wordsInCorpus is sparse
  - Only numDocs $\times$ (numDistinctWordsPerDoc)^2 execs of inner loop
  - But Python is an interpreted language!!

Imp 2: Vector-Based with Loop over Docs

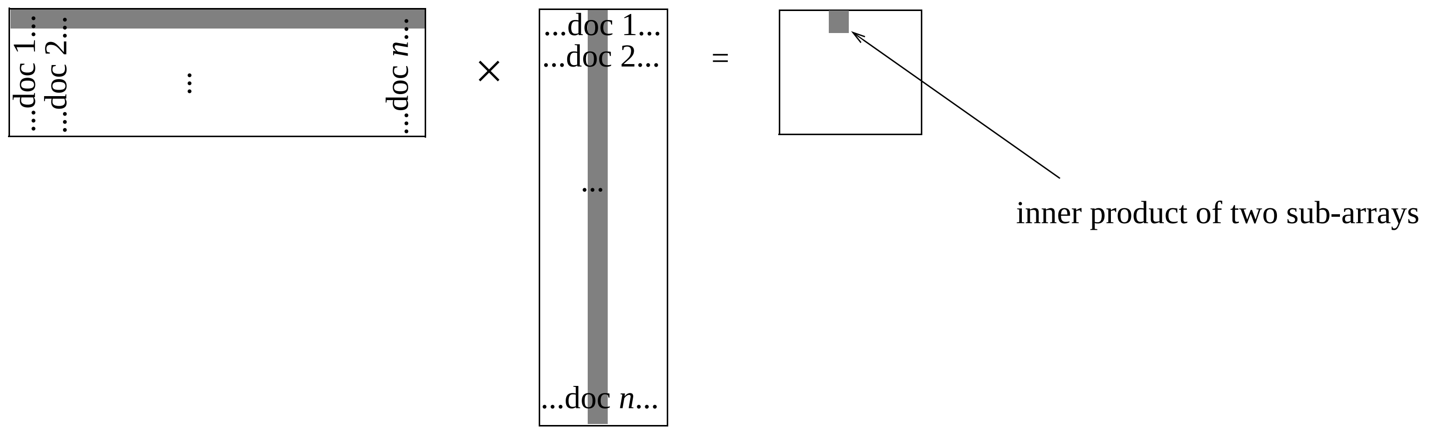
- Given a 1-d array `array` = [0, 0, 3, 1, 0, 1...]...
  - — The *outer product* of `array` with itself creates a 2-d matrix
  - — Where *i*th row is `array[i]` $\times$ `array`
  - — So if an `array` gives number of occurs of each word in a doc...
  - — And we *clip* `array` so [0, 0, 3, 1, 0, 1...] becomes [0, 0, 1, 1, 0, 1...]
  - — Then take outer product of `array` with itself...
  - — Entry at pos `[i, j]` is number of co-occurs of dictionary words `i`, `j` in doc
- Note:
  - — `np.outer (arrayOne, arrayTwo)` is outer product of arrays
  - — `np.clip (array, low, high)` clips all entries to max of `high`, min of `low`

Imp 3: Pure Vector-Based



- Note that after matrix multiply
  - — Entry at pos `[i, j]` is inner product of row `i` from LHS, col `j` from RHS
  - — So if row `i` is number of occurs of word `i` in every doc
  - — And if col `j` is number of occurs of word `j` in every doc
  - — Entry at pos `[i, j]` is number of co-occurs of words `i, j`
  - — Suggests a super-efficient algorithm

Imp 3: Pure Vector-Based



- Some notes:
  - `np.transpose (array)` computes transpose of matrix in `array`
  - `np.dot (array1, array2)` computes dot product of 1-d arrays, matrix multiply of 2-d

Time to Write Some Code

- Go to `cmj4.web.rice.edu/CoOccur.html`
  - Complete the three tasks
  - Pay special attention to the running times!

Aggregations Over Arrays

- In statistical/data analytics programming...
  - Tabulations, max, min, etc. over NumPy arrays are ubiquitous
- Key operation allowing this is sum
  - Ex: `array` is [1, 2, 3, 4, 5]
  - `array.sum ()` is 15
- Can sum along dimension of higher-d array.
  - Ex: `array` is [[1, 2, 3, 4, 5], [1, 2, 3, 4, 5], [1, 2, 3, 4, 5]]
  - `array.sum (0)` is [3, 6, 9, 12, 15]
  - `array.sum (1)` is [15, 15, 15]

Subscripting Arrays

- To compute various tabulations, need to access subarrays
  - Ex: `array` is [[1, 2, 3, 4, 5], [2, 3, 4, 5, 6], [3, 4, 5, 6, 7]]
  - `array[1:,]` or `array[1:]` is [[2, 3, 4, 5, 6], [3, 4, 5, 6, 7]]
  - Why? Gets rows 1, 2, 3, ...
  - `array[2:3,]` or `array[2:3]` is [[3, 4, 5, 6, 7]]
  - Why? Gets row 2
  - `array[0:2,]` or `array[0:2]` is [[1, 2, 3, 4, 5], [2, 3, 4, 5, 6]]
  - `array[:,1:3]` is [[2, 3], [3, 4], [5, 6]]
  - `array[:,np.array([1,2])]`is also [[2, 3], [3, 4], [5, 6]]

Other Useful Tabulaton Functions

- To compute max:
  - Ex: `array` is [[10, 2, 3, 4, 5], [2, 13, 4, 5, 6], [3, 4, 5, 6, 7]]
  - `array.max()` is 13
  - Can tabulate over dimensions
  - `array.max(0)` is [10, 13, 5, 6, 7]
  - `array.max(1)` is [10, 13, 7]
- To compute the position of the max:
  - Ex: `array` is [[10, 2, 3, 4, 5], [2, 13, 4, 5, 6], [3, 4, 5, 6, 7]]
  - `array.argmax()` is 6
  - `array.argmax(0)`is [0, 1, 2, 2, 2]

Useful Array Creation Functions

- To create a 2 by 5 array, filled with 3.14
  - `np.full((2, 5), 3.14)`
- To create a 2 by 5 array, filled with zeros
  - `np.zero((2, 5))`
- To create an array with odd numbers thru 10
  - `np.arange(1, 11, 2)`gives [1, 3, 5, 7, 9]
- To tile an array
  - `np.tile (np.arange(1, 11, 2), (1, 2)` gives [1, 3, 5, 7, 9, 1, 3, 5, 7, 9]
  - `np.tile (np.arange(1, 11, 2), (2, 1)` gives [[1, 3, 5, 7, 9], [1, 3, 5, 7, 9]]

Time to Write The Day’s Last Code

- Go to `cmj4.web.rice.edu/Subarrays.html`